Writing in Mathematics: An Investigation of the Impact on Mathematical Reasoning Skills

Lindy Crawford, Ph.D., Kristina Higgins, Ph.D., Jacqueline Huscroft-D'Angelo, Ph.D., Texas Christian University

Introduction

- Reasoning is a critical aspect of any mathematics curriculum and is an area in which students with Learning Disabilities (LD) experience difficulty (Bressette, 2010).
- Implementing writing into mathematics has become a valuable tool to help students express mathematical reasoning, interact with peers, clarify ideas, and reflect on the processes involved in mathematics (Cooper, 2012; Morris, 2006; National Council of Teachers of Mathematics, 2000; Pearson, 2010).
- The use of technologies has become a strategy to promote writing in mathematics (Cooper, 2012; Zemelman, Daniels, & Hyde, 2005).
- However, little empirical evidence exists on how the use of writing facilitated through technology affects the mathematical reasoning of students, particularly for students with disabilities.

Purpose

Examine how mathematical reasoning changes over time for students with LD when provided access and training on the use of a digital writing environment within an online supplemental mathematics curriculum.

Research Questions

1. What are the mathematical reasoning skills of students with LD prior to, and following the use of, a digital writing environment?
2. To what extent and in what ways does the digital writing environment affect the quality of mathematical reasoning for students with LD over time?

Method

Participants

This study included a small sample ($N = 13$) of students with LD in grades 3 (46.2%) and 4 (53.8%) from two private schools in Dallas, TX. The majority of participants were Caucasian ($n = 10$), followed by African-American ($n = 2$), and Asian ($n = 1$). Just over half of the sample ($n = 7$) was male. Four of the students had primary LD in reading, three in writing, three in math, and three were identified as general LD.

Intervention

Subjects participated in an intervention focused on writing in mathematics through the use of a digital writing environment. Participants completed eight grade-appropriate lessons from a supplemental online mathematics program (Math Learning Companion; MLC). Over the eight lessons, students were provided with explicit instruction on how and when to record their thoughts on a digital notepad and a peer-mediated wall or blogging tool. New writing demands were placed on students every two lessons. Figure 1 describes the intervention in detail.

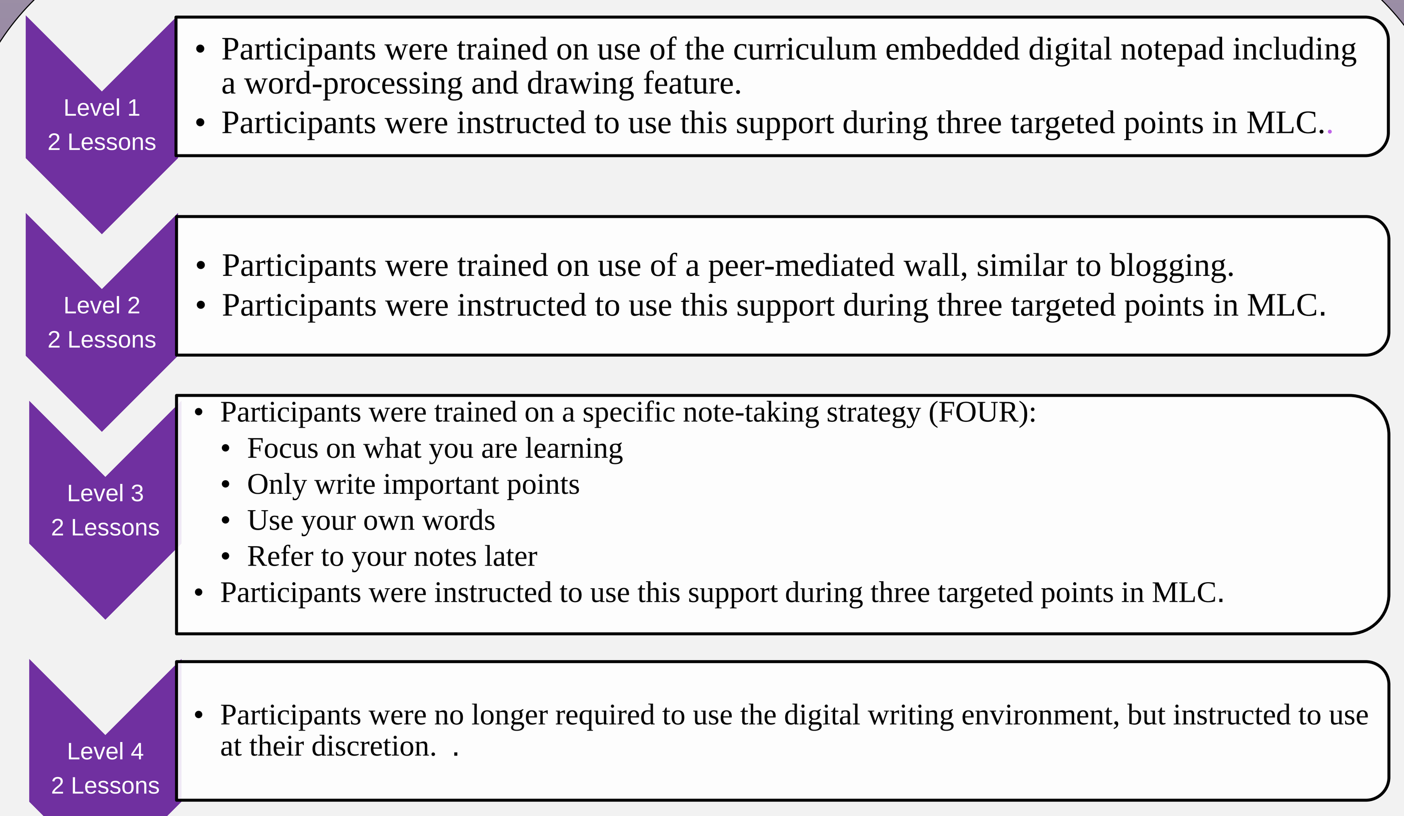

Figure 1. *Intervention Levels for Writing and Mathematical Reasoning*

Measures

Math Reasoning Inventory (MRI; Burns, 2012) - An online formative assessment primarily designed to serve as a tool for conducting face-to-face interviews that focus on core numerical reasoning strategies and understanding. Only the Whole Numbers portion of the MRI was used for the purpose of this study. This consisted of 10 items in which students were provided a problem, asked to answer the problem without the use of pencil and paper, and then asked to provide an explanation of how they obtained their answer. The reliability of the MRI for the Whole Numbers test is Cronbach's Alpha of .81 and person separation reliability statistics of .74 (Bernbaum-Wilmot, 2012).

Data Analysis

Prior to analysis of the Math Reasoning Inventory (MRI) three distinct steps were implemented to code the MRI data for analyses:

1. Developed coding dictionary for three sections of MRI (Answering, Explanation, and Reasoning).
2. Established reliability between coders on 11% of the sample with an average of 95.8% reliability, all disagreements were discussed until a consensus was reached.
3. Independently coded MRI

Table 1
Examples of Different Types of Student Reasoning on MRI

Type of Reasoning	Question	Student Answer	Student Explanation	Student Reasoning
No attempt at reasoning	7000/70	Did not answer	N/A	I don't know, not big on division in my head. It's hard for me.
Guess attempt, incorrect	If 20x15=300 what does 21x15 = ___	Incorrect (305)	Guessed, did not explain, or gave faulty explanation	Doing it vertical, I knew 5x5=1 and I knew 2x5=10 so you put down your zero
Guess attempt, correct	99+17	Correct	Used standard algorithm to add	I just added them in my head
Partial attempt, reasoning breakdown	100-18	Incorrect (81)	Gave other reasonable explanation	When you think about it, it can't be in the 90s because 18 is more than 10, but it can't be in the 100s because you are taking away, so it has to be in the 80s
Complete reasoning, calculation error	99+17	Incorrect (117)	Used standard algorithm to add	9+7=16 and you put your 1 by the other 9 and that makes 10 and 10+1 would equal 11 and you put the one in the other place and that makes it 117
Complete reasoning, correct	99+17	Correct	Used other method specific to problem	I split the 9 and the 1 up and the equals 10 and I put the 9 in my head and I counted 7 more
Entirely wrong process	If 20x15=300 what does 21x15 = ___	Incorrect (215)	Guessed, did not explain, or gave faulty explanation	I take the 1 away and put the 2 in front of the 15

Note. Student Answer = correctness of the answer and fell into four categories: correct, incorrect, self-corrected, or did not answer. *Student Explanation* = how students solved the problem fell into four broad categories: used the standard algorithm, used another method specific to the problem, gave another reasonable explanation, or guessed. *Student Reasoning* = thought process communicated verbally was coded into seven categories: no attempt at reasoning, guess attempt but incorrect answer, guess attempt and correct answer, partial attempt but had a reasoning breakdown, complete reasoning with a calculation error, complete reasoning and correct answer, or entirely wrong process.

Results

Table 2
Observed and Expected Frequencies of MRI Responses for Students with LD (N = 13)

	Pretest		Posttest	
	Observed Frequency	Expected Frequency	Observed Frequency	Expected Frequency
Correct	21	37	31	47
Incorrect	93	77	74	65
Self-corrected	2	3	1	4
Did not answer	14	13	24	14
Used standard algorithm	28	29	18	14
Other method specific to problem	15	30	18	29
Gave other reasonable explanation	4	10	8	14
Guessed	80	58	63	50
No attempt at reasoning	2	3	14	6
Guess attempt, incorrect	24	19	12	14
Guess attempt, correct	3	2	2	2
Partial attempt, reasoning breakdown	28	21	21	18
Complete reasoning, calculation error	20	22	18	18
Complete reasoning, correct	20	38	30	46
Entirely wrong process	18	11	17	9

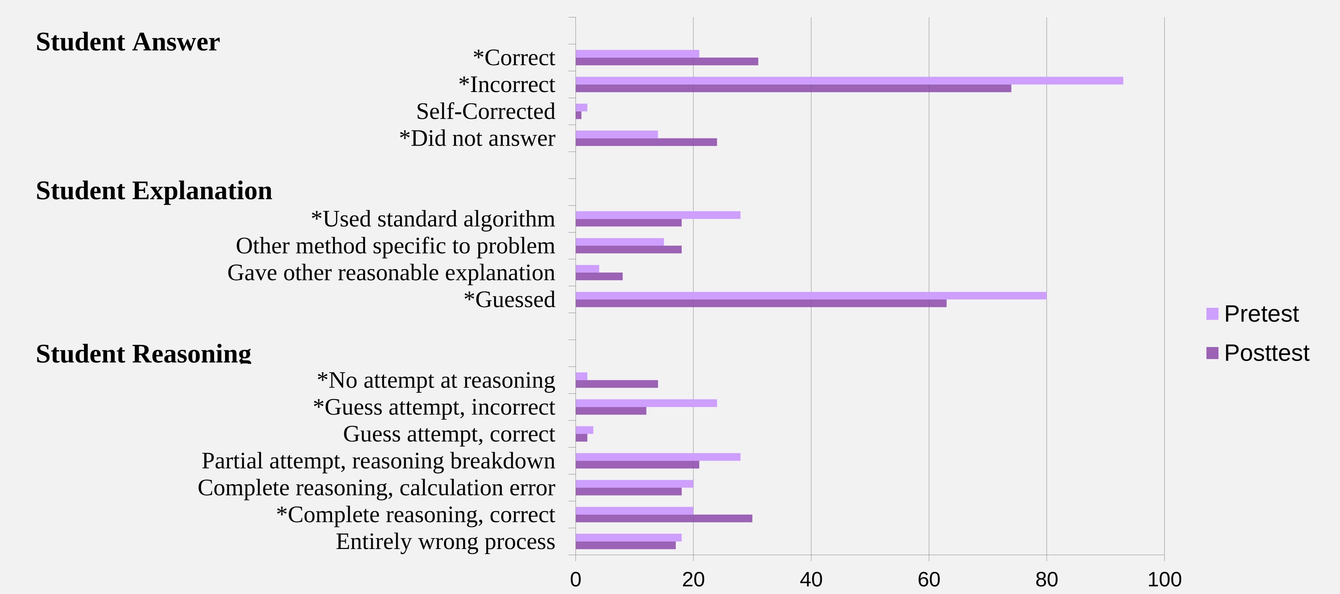

Figure 2.
Changes in Mathematical Reasoning from Pretest to Posttest for Students with LD
Reflects differences between the observed frequencies from pre to posttest that have a standardized residual > 1.96

Student Answer: $\chi^2 = 29.794$, df = 3, $p < .001$; ES $\phi = .479$
Student Explanation: $\chi^2 = 32.058$, df = 3, $p < .001$, ES $\phi = .555$
Student Reasoning: $\chi^2 = 77.109$, df = 6, $p < .001$, ES $\phi = .861$

Conclusions

- Students with LD made gains in reasoning and correct responses from pre/post test.
- At posttest students responded correctly more often, provided a standard algorithm less often, and gave an additional method specific to the problem more often.
- At posttest students were less likely to have a reasoning breakdown or calculation error, and were more likely to have a complete reasoning process and get the answer correct.

Implications

Incorporating writing in mathematics through technology and training students on the use of note-taking, recording thoughts or processes, and engaging in discussions with peers around mathematically relevant topics are beneficial for the mathematical reasoning skills of students with LD.